
ABSTRACT

This project work expresses the difference between the structures with and without damping material. The effect of damping on the performance of isotropic (steel) and orthotropic (Carbon Epoxy) structures is to be analysed by using Finite Element Analysis. The values of damping factor, fundamental natural frequency and the static deflection for Steel Shaft, Carbon Epoxy Shaft and are to be compared with and without viscoelastic polymer (Rubber).

A new composite damping material is to be studied, which consists of a viscoelastic matrix and high elastic modulus fiber inclusions. This fiber-enhanced viscoelastic-damping polymer is intended to be applied to lightweight flexible structures as a surface treatment for passive vibration control. A micro mechanical model is to be established and closed form expressions for the effective storage and loss properties of the damping material are to be derived. An optimal relation between design parameters, such as the length, diameter, spacing, and Young's modulus of fibers and the shear modulus of viscoelastic matrix, is to be derived for achieving maximum damping performance. The characteristic value for the maximum value of Loss modulus is to be found out for the different values of matrix loss factors.

KEYWORDS: Viscoelastic polymers, Static, Model, Transient Dynamic Analysis.

INTRODUCTION

Composite materials are those containing more than one bonded material, each with different material properties. The major advantages of composite materials are that they have a high ratio of stiffness to weight and strength to weight. A principal advantage of composite materials lies in the ability of the designer to tailor the material properties to the application

Viscoelastic material

A Viscoelastic material sometimes is called material with memory. This implies that a Viscoelastic material's behavior depends not only on the current loading conditions, but also on the loading history. They are characterized by possessing both viscous and elastic behavior. Figure 1.2 shows how various types of materials behave in the time domain.

A purely elastic material is one in which all the energy stored in the sample during loading is returned when the load is removed. As a result, the stress and strain curves for elastic materials move completely in phase. For elastic materials, Hooke's Law applies, where the stress is proportional to the strain, and the modulus is defined as the ratio of stress to strain.

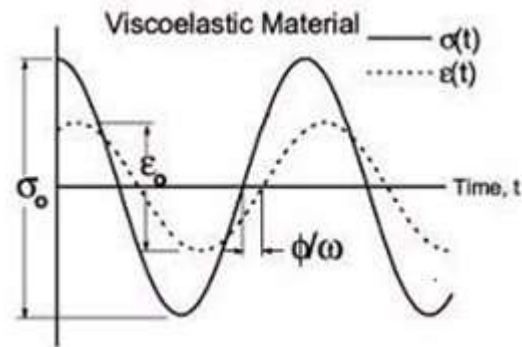


Fig 1.2 Stress Strain relationship of Viscoelastic Material w.r.t Time

LITERATURE SURVEY

Viscoelastic damping materials add passive damping to structures by dissipating vibration strain energy and generate heat energy. The incorporation of damping materials in advanced composite materials offers the possibility of highly damped, light weight structural components that are vibration resistant. Concurring viscoelastic damping materials in composites has shown to be successful in greatly increasing the damping of composite structures. The damping performance, however, is often not as high in cocured composites as in secondarily bonded composites, where the damping material does not undergo the cure process. Substituting composite structures for conventional metallic structures has many advantages because of higher specific stiffness and specific strength of composite materials. The fiber enhanced viscoelastic damping polymer is intended to be applied to lightweight flexible structures as a surface treatment for passive vibration control. A desirable packing geometry for the composite material is proposed, which is expected to produce maximum shear strain in the viscoelastic damping matrix. A general method for modeling material damping in dynamical systems is presented and it is primarily concerned with a dissipation model based on viscoelastic assumptions. Different numerical approaches for modeling and analyzing the behavior of structures having constrained layer damping. Two numerical studies are presented that reveal the accuracy limits of the different finite element modeling approaches for additive and integrally damped plate type structures. Now through the use of improved computational – based approaches (i.e. finite element method) along with the availability of reliable damping materials with accurate thermal and dynamic property characterizations, it is possible to incorporate damping treatments as part of the initial structural design process, thereby virtually eliminating sharp resonant peaks. Cylindrical shells with a constrained damping layer treatment are studied using three theories. Constrained layer damping in structures is a very popular method to control resonant amplitudes of vibration. Shells of revolution (e.g., cylindrical and conical) find wide application in the aerospace industry. An efficient method is described for finite element modeling of three-layer laminates containing a viscoelastic layer. Modal damping ratios are estimated from undamped normal mode results by means of the Modal Strain Energy (MSE) method. The solution for a radially simply supported shell has been obtained and the procedure for determining the damping effectiveness in terms of the system loss factor for all families of the modes of vibration in a multilayered shell with elastic and viscoelastic layers is reported. Approximately 85% of the passive damping treatments in actual applications are based on viscoelastic materials. The solution for the vibration and damping analysis of a general multilayered cylindrical shell consisting of an arbitrary number of orthotropic material elastic and viscoelastic layers with simply supported end conditions has been reported.

DESCRIPTION OF PROBLEM

The torque transmission capability of the propeller shaft for passenger cars, small trucks, and vans should be larger than 3,500 Nm and fundamental natural bending frequency of the propeller shaft should be higher than 6,500 rpm to avoid whirling vibration. The outer diameter of the propeller shaft should not exceed 100 mm due to space limitations. The propeller shaft of transmission system is designed for following specified design requirements as shown in Table 4. 1. Due to space limitations the outer diameter of the shaft is restricted to 90.24 mm. The one-piece hollow composite drive shaft for rear wheel drive automobile should satisfy three design specifications, such as static torque transmission capability, torsional buckling capacity and the fundamental natural bending frequency. For

given specification, the damping factor for Steel, carbon Epoxy are to be calculated and compared with and without damping material (Rubber).

Table 4.1 Problem Specification

Sl. No.	Parameter	Notation	Units	Value
1.	Torque	T	N-m	3500
2.	Max Speed	N	RPM	6500
3.	Length	L	m	1.250

Table 4.2 Material Properties.

Sl. No.	Properties	Units	Steel	Carbon Epoxy	Rubber
1	Young's Modulus E11	N / m ²	2.068e ¹¹	1.34 e ¹¹	3.1027e ⁶
2	Young's Modulus E22	N / m ²	2.068e ¹¹	7 e ⁹	3.1027e ⁶
3	Density	kg / m ³	7830	1600	2.466
4	Poisson Ratio	-	0.3	0.3	0.49
5	Shear Modulus G	N / m ²	-	5.8e ⁹	1.379e ¹¹

A new composite damping material is to be studied, which consists of a viscoelastic matrix and high elastic modulus fiber inclusions. This fiber-enhanced viscoelastic-damping polymer is intended to be applied to lightweight flexible structures as a surface treatment for passive vibration control. A micro mechanical model is to be established and closed form expressions for the effective storage and loss properties of the damping material are to be derived. An optimal relation between design parameters, such as the length, diameter, spacing, and Young's modulus of fibers and the shear modulus of viscoelastic matrix, is to be derived for achieving maximum damping performance.

STATIC ANALYSIS

Static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects, such as those caused by time-varying loads. A static analysis, however, includes steady inertia loads (such as gravity and rotational velocity), and time-varying loads that can be approximated as static equivalent loads (such as the static equivalent wind and seismic loads commonly defined in many building codes).

Static Analysis of Steel Shaft without viscoelastic Damping Material by Beam and Shell Element

In this part static deflection of the steel shaft is calculated and compared with ANSYS results. The specification for the shaft is given in the Table 5.1. For calculating the deflection, the cantilever boundary condition is taken by considering its self weight.

Table 5.1 Specification of steel shaft

Sl. No.	Parameters	Values
1	Outer Diameter	0.09024 m
2	Thickness	2.1 e ⁻³

Steel Shaft without Damping Material for Shell 99

In this case shell element is taken to calculate the deflection value for steel shaft. Here Shell element is taken due to specify the number of layers to include the damping polymer. Here steel shaft without damping material is considered and specifications are tabulated in table 5.1.

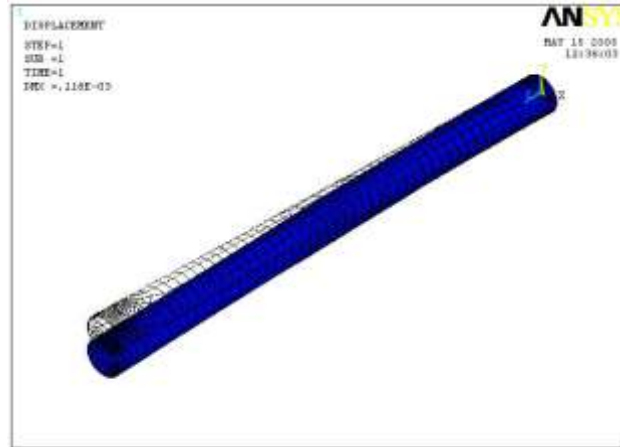


Fig 5. 2 Static Deflection for Steel Shaft

Using ANSYS 7.0 the deflection value is calculated. The value is 0.116×10^{-3} m. The deformed shape of the shaft is shown in the Fig 5.2.

Steel Shaft with Damping Material

In this type a damping material (i.e.) Rubber is inserted between the two layers of shaft and the deflection value is calculated using ANSYS. The specification of the shaft with damping material is shown in the Table 5.3

Table 5. 3 Specifications for Steel Shaft with Rubber

Sl. No.	Parameters	Values
1	Outer Diameter	0.09024 m
2	Thickness of each layer	1.05×10^{-3} m
3	Number of layers	3
4	Damping Material	Rubber
5	Element	Shell 99

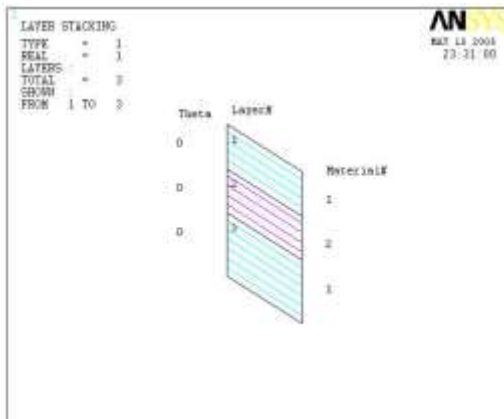


Fig 5.3 Stacking Sequence for Steel Shaft with Rubber

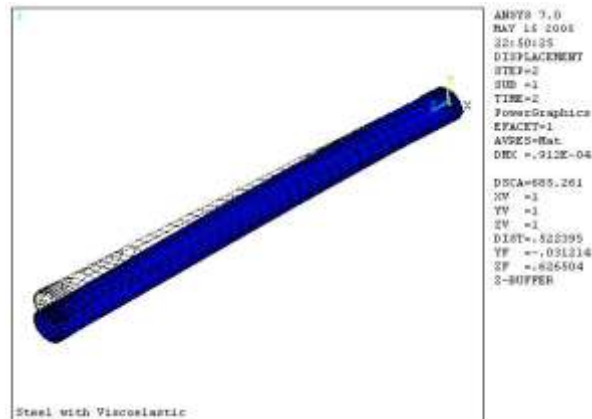


Fig 5.4 Deflection of Steel Shaft with Rubber

Using ANSYS 7.0 the deflection value is calculated. The value is 0.912×10^{-4} m. The deformed shape of the shaft is shown in the Fig 5.4.

Carbon Epoxy Shaft without Damping Material

In this case Carbon Epoxy shaft is modeled with 13 layers by considering the shell element. The specifications are shown in the Table 5.4

Table 5.4 Specification for Carbon Epoxy Shaft

Sl. No.	Parameters	Values
1	Outer Diameter	0.09024 m
2	Thickness of each layer	1.5×10^{-4} m
3	Number of layers	13
4	Element	Shell 99

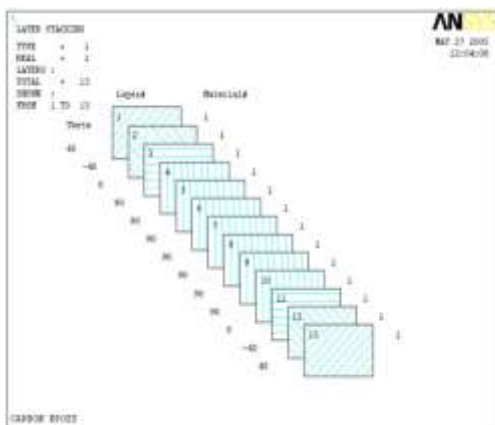


Fig 5.5 Stacking Sequence for Carbon Epoxy Shaft

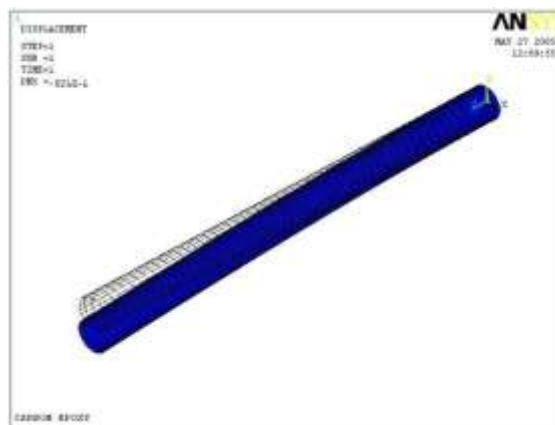


Fig 5.6 Static Deflection for Carbon Epoxy Shaft

Using ANSYS 7.0 the deflection value is calculated. The value is 0.824×10^{-4} m. The deformed shape of the shaft is shown in the Fig 5.6.

Carbon Epoxy Shaft with Damping Material

In this case Carbon Epoxy shaft is modeled with damping material (Rubber) and it is incorporated in between the layers. The specification of the shaft is shown in the Table 5.5.

Table 5.5 Specification for Carbon Epoxy Shaft with Rubber

Sl. No.	Parameters	Values
1	Outer Diameter	.09024 m
2	Thickness of each layer	1.5×10^{-4} m
3	Number of layers	14
4	Damping Material	Rubber
5	Element	Shell 99

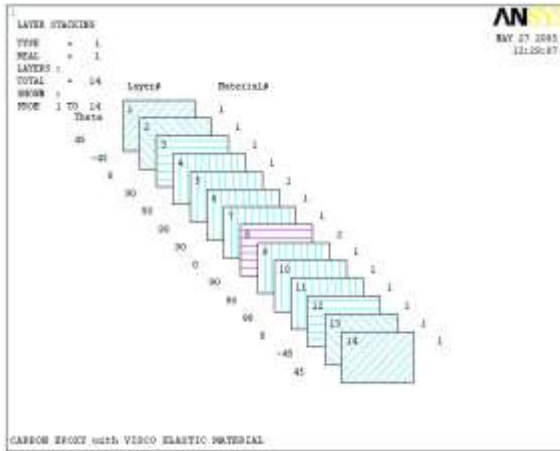


Fig 5.7 Stacking Sequence for Carbon Epoxy Shaft

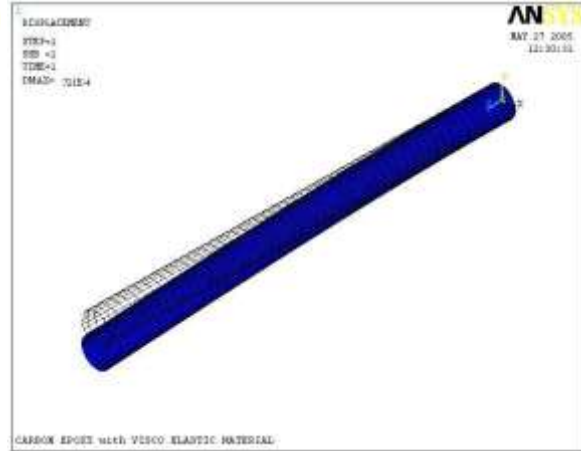


Fig 5.8 Static Deflection for Carbon Epoxy Shaft with Rubber

The stacking sequence of the Carbon Epoxy shaft with damping material (Rubber) is shown in the fig 5.7. Here the 7th layer is the rubber. Using ANSYS 7.0 the deflection value is calculated. The value is 0.712×10^{-4} m. The deformed shape of the shaft is shown in the Fig 5.8.

MODAL ANALYSIS

Any physical system can vibrate. The frequencies at which vibration naturally occurs, and the modal shapes which the vibrating system assumes are properties of the system, and can be determined analytically using Modal Analysis.

Modal analysis is the procedure of determining a structure's dynamic characteristics; namely, resonant frequencies, damping values, and the associated pattern of structural deformation called mode shapes. It also can be a starting point for another, more detailed, dynamic analysis, such as a transient dynamic analysis, a harmonic response analysis, or a spectrum analysis.

Modal analysis in the ANSYS family of products is a linear analysis. Any nonlinearities, such as plasticity and contact (gap) elements, are ignored even if they are defined. Modal analysis can be done through several mode extraction methods: subspace, Block Lanczos, Power Dynamics, Reduced, Unsymmetric and Damped. The damped method allows you to include damping in the structure.

Modal Analysis of Steel Shaft using Beam Element without Rubber

Consider the free-body diagram of an element of a beam shown in figure. Where $M(r, t)$ is the bending moment, $V(r, t)$ is the shear force and $f(r, t)$ is the external force per unit length of the beam.

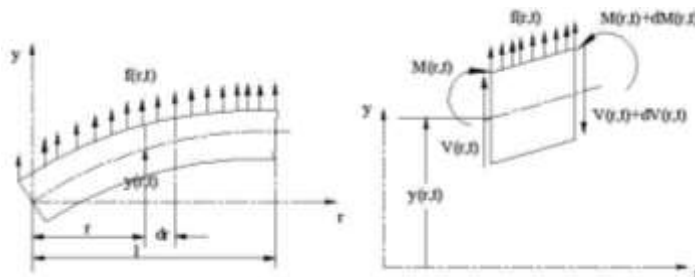


Fig 6.1 Beam in Bending

Euler -Bernoulli beam equations with external force, external moment is given by following equations.

$$EI \frac{\partial^4 y(r,t)}{\partial r^4} + \rho A \frac{\partial^2 y(r,t)}{\partial t^2} = f(r,t) \quad \dots\dots\dots (1)$$

Using Assumed modes approach, by solving the above equation the Eigen values and mode shapes can be calculated. Using the variable separable method $y(r, t)$ may be expressed by following equations.

$$y(r,t) = \sum_{i=1}^{\infty} \Phi_i(r) q_i(t) \quad \dots\dots\dots (2)$$

$$EI \sum_{i=1}^{\infty} \Phi_i''''(r) q_i(t) + \rho A \sum_{i=1}^{\infty} \Phi_i(r) q_i''(t) = C_a \frac{\partial^2 V_a(r,t)}{\partial r^2} \quad \dots\dots\dots (3)$$

$$EI \int_0^L \sum_{i=1}^{\infty} \Phi_i''''(r) q_i(t) dr + \rho A \int_0^L \sum_{i=1}^{\infty} \Phi_i(r) q_i''(t) dr = C_a \int_0^L \frac{\partial^2 V_a(r,t)}{\partial r^2} dr \quad \dots\dots\dots (4)$$

By multiplying $\Phi_i(r)$ on both sides then apply the Orthogonality principle.

$$EI \int_0^L \sum_{i=1}^{\infty} \Phi_i''''(r) q_i(t) \Phi(r) dr + \rho A \int_0^L \sum_{i=1}^{\infty} \Phi_i(r) q_i''(t) \Phi(r) dr = C_a \int_0^L \frac{\partial^2 V_a(r,t)}{\partial r^2} \Phi(r) dr \quad \dots\dots (5)$$

$$\Phi_i''''(r) - \left(\frac{EI\omega^2}{\rho A} \right) \Phi_i(r) = 0 \quad \dots\dots\dots (6)$$

$$\Phi_i^4(r) - (\lambda^4) \Phi_i(r) = 0 \quad \dots\dots\dots (7)$$

The complementary solution of the above equation is given by

$$\Phi_i(r) = A_i \sin(\lambda_i r) + B_i \cos(\lambda_i r) + C_i \sinh(\lambda_i r) + D_i \cosh(\lambda_i r) \quad \dots\dots\dots (8)$$

Boundary conditions of cantilever beam,

At the fixed end: $y(0,t) = 0, EI \frac{\partial y(r,t)}{\partial r} = 0 \quad \dots\dots\dots (9)$

At the free end: $EI \frac{\partial y^2(L,t)}{\partial r^2} = 0, EI \frac{\partial y^3(L,t)}{\partial r^3} = 0 \quad \dots\dots\dots (10)$

Applying these boundary conditions the solution of differential equation transformed

to $\Phi_i(r) = L \left[\cosh \lambda_i r - \cos \lambda_i r - \left(\frac{\cosh \lambda_i L + \cos \lambda_i L}{\sinh \lambda_i L + \sin \lambda_i L} \right) \times (\sinh \lambda_i r - \sin \lambda_i r) \right] \dots (11)$

Shear force at free end is zero, by applying this boundary condition in the above equation the above equation converted to following equation.

$$1 + \cos \lambda_i L \cosh \lambda_i L = 0 \quad \dots\dots\dots (12)$$

From the solution of the equation the λ_i value can be calculate. The natural frequency of the system is given by substituting λ_i can be calculated.

$$\omega_i = \sqrt{\frac{EI}{\rho A}} \lambda_i^2 \quad \dots\dots\dots (13)$$

Table 6.1 Modal Frequencies for Steel Shaft using BEAM 3

Mode Shapes	Theoretical Value (HZ)	Analytical Value Using ANSYS 7.0 (HZ)
1	58.29	57.289
2	359.68	355.98
3	1007.04	983.61
4	1972.94	1892
5	3260	3112.1

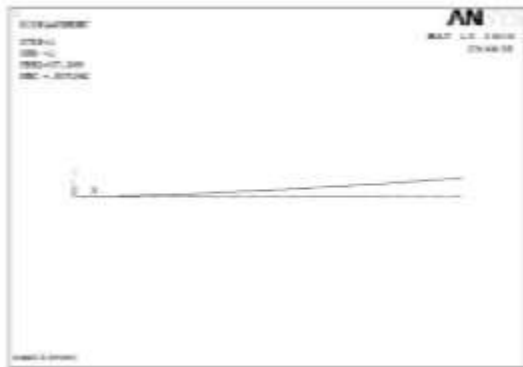


Fig 6.2 Modal Analysis for Steel Shaft using Beam 3

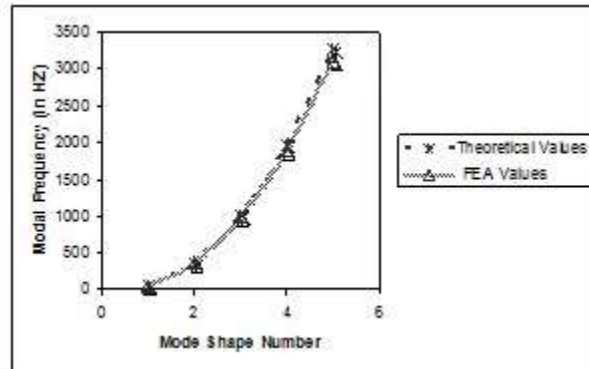


Fig 6.3 Comparison of Results

The fundamental natural frequency of Steel shaft using Beam 3 element is shown in the Fig 6.2. The value of the frequency is 57.289 Hz. The theoretical and FEA values are compared and shown in the Fig 6.3.

Modal Analysis of Steel Shaft using Shell Element without Rubber and with Rubber

In this case Steel shaft is modeled using Shell 99. The specifications used are same as in the Static Analysis.

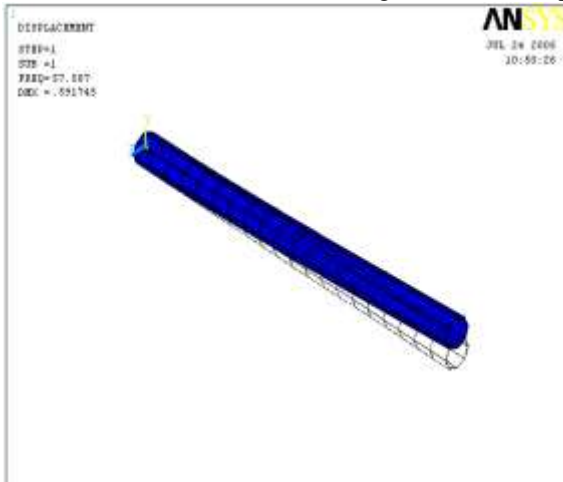


Fig 6.4 Modal Analysis for Steel Shaft using Shell 99 Rubber

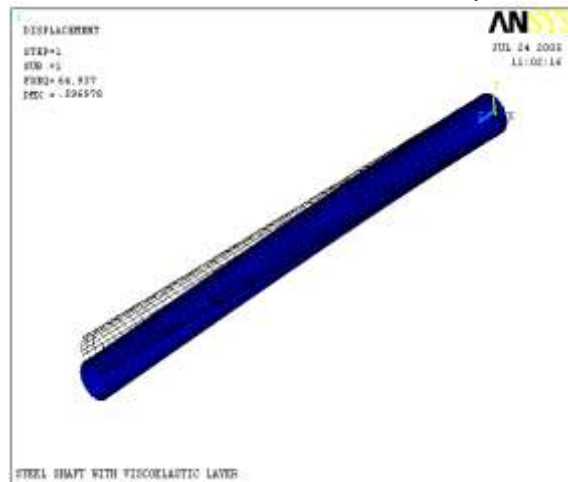


Fig 6.5 Modal Analysis for Steel Shaft with Rubber

The fundamental natural frequency of the steel shaft using Shell 99 is shown in the Fig 6.4. The value is 57.587 Hz. The fundamental natural frequency of the Steel Shaft with Rubber is shown in the Fig 6.5. The value is 64.937 Hz.

Modal Analysis Of Carbon Epoxy Shaft Without rubber and With Rubber

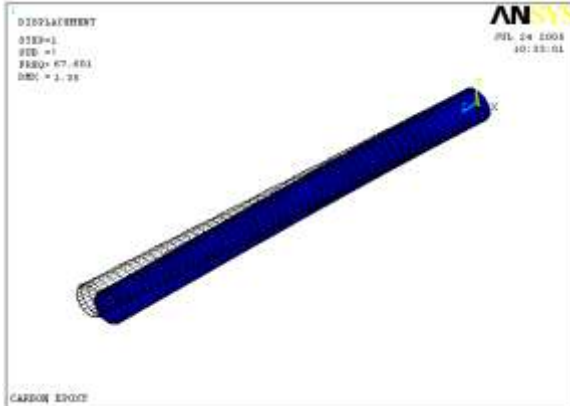


Fig 6.6 Modal Analysis for Carbon Epoxy Shaft

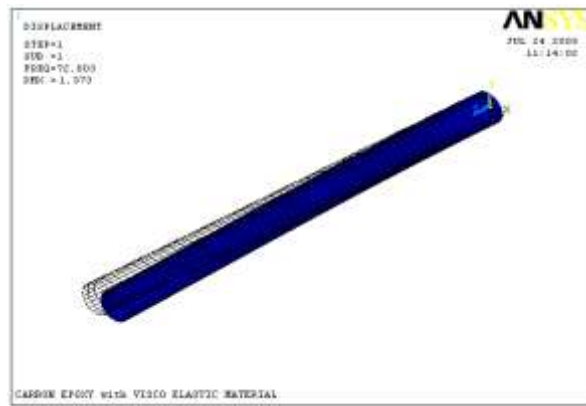


Fig 6.7 Modal Analysis for Carbon Epoxy Shaft with Rubber

The fundamental natural frequency of the Carbon Epoxy Shaft is shown in the Fig 6.6. The value is 67.601 Hz. The fundamental natural frequency of the Carbon Epoxy Shaft with is shown in the Fig 6.7. The value is 72.443 Hz

TRANSIENT DYNAMIC ANALYSIS

Transient dynamic analysis is a technique used to determine the dynamic response of a structure under a time-varying load. The time frame for this type of analysis is such that inertia or damping effects of the structure are considered to be important. Cases where such effects play a major role are under step or impulse loading conditions, for example, where there is a sharp load change in a fraction of time. If inertia effects are negligible for the loading conditions being considered, a static analysis may be used instead.

It should be noted that a transient analysis is more involved than a static or harmonic analysis. It requires a good understanding of the dynamic behavior of a structure. Therefore, a modal analysis of the structure should be initially performed to provide information about the structure's dynamic behavior.

In ANSYS, transient dynamic analysis can be carried out using 3 methods.

- **The Full Method:** This is the easiest method to use. All types of non-linearities are allowed. It is however very CPU intensive to go this route as full system matrices are used.
- **The Reduced Method:** This method reduces the system matrices to only consider the Master Degrees of Freedom (MDOFs). Because of the reduced size of the matrices, the calculations are much quicker. However, this method handles only linear problems (such as our cantilever case).
- **The Mode Superposition Method:** This method requires a preliminary modal analysis, as factored mode shapes are summed to calculate the structure's response. It is the quickest of the three methods, but it requires a good deal of understanding of the problem at hand.

In this project the Reduced Method is used for conducting the transient analysis. Usually one need not go further than reviewing the Reduced Results. However, if stresses and forces are of interest than, we would have to Expand the Reduced Solution.

Transient Dynamic Analysis of Steel Shaft without damping material by beam element and shell element

In this part Transient Dynamic analysis of the shaft is calculated using the same specifications as given in the static analysis. Transient Dynamic analysis is needed because the output of this analysis is used in calculating the damping factor using Log Decrement method.

Transient Dynamic Analysis of Steel Shaft using Beam Element

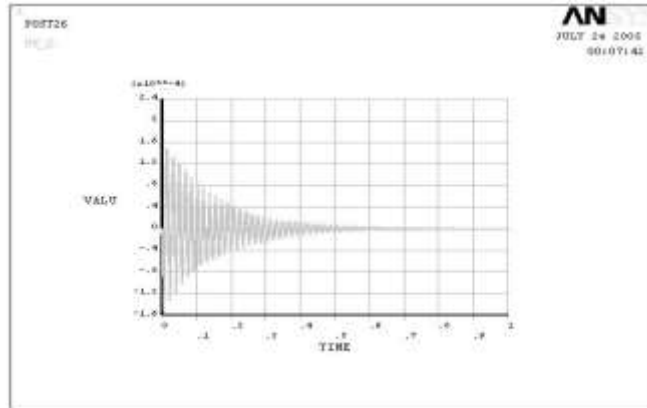


Fig 7.1 Transient Dynamic Analysis of Steel Shaft using Beam 3

The Transient Dynamic Analysis of Steel Shaft using Beam 3 Element is shown in the Fig 7.1. From the above Fig 7.1 damping factor is calculated using Log Decrement method and the value is 0. 01396

Transient Dynamic Analysis of Steel Shaft using Shell Element without and with Rubber

In this case Steel shaft is modeled using Shell 99. The specifications used are same as in the Static Analysis.

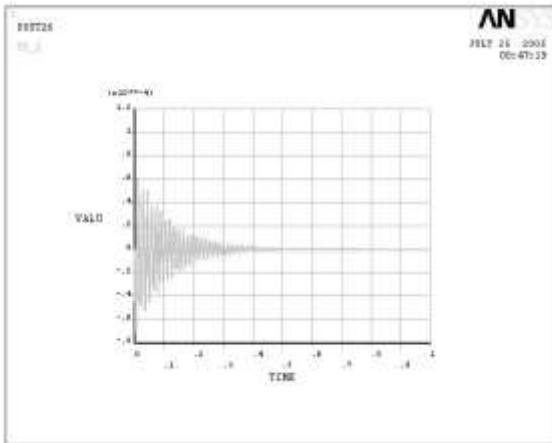


Fig 7.2 Transient Analysis for Steel Shaft without rubber

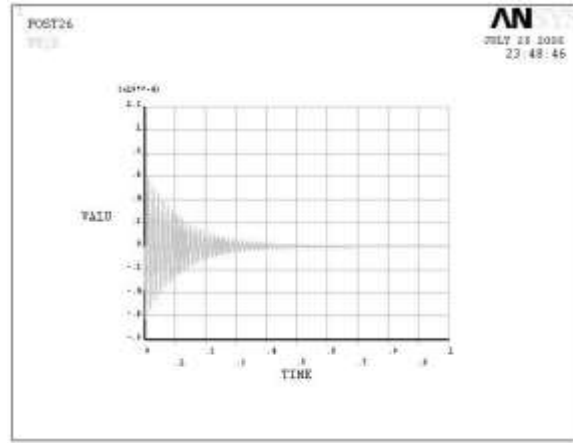


Fig 7.3 Transient Analysis for Steel Shaft using Rubber

The Transient Dynamic Analysis of Steel Shaft using Beam 3 Element is shown in the Fig 7.2. From the above Fig 7.2 damping factor is calculated using Log Decrement method and the value is 0. 016766. The Transient Dynamic Analysis of Steel Shaft using Shell 99 with Rubber is shown in the Fig 7.3. From the above Fig 7.3 damping factor is calculated using Log Decrement method and the value is 0. 0195.

Transient Dynamic Analysis of Carbon Epoxy Shaft without and with Rubber

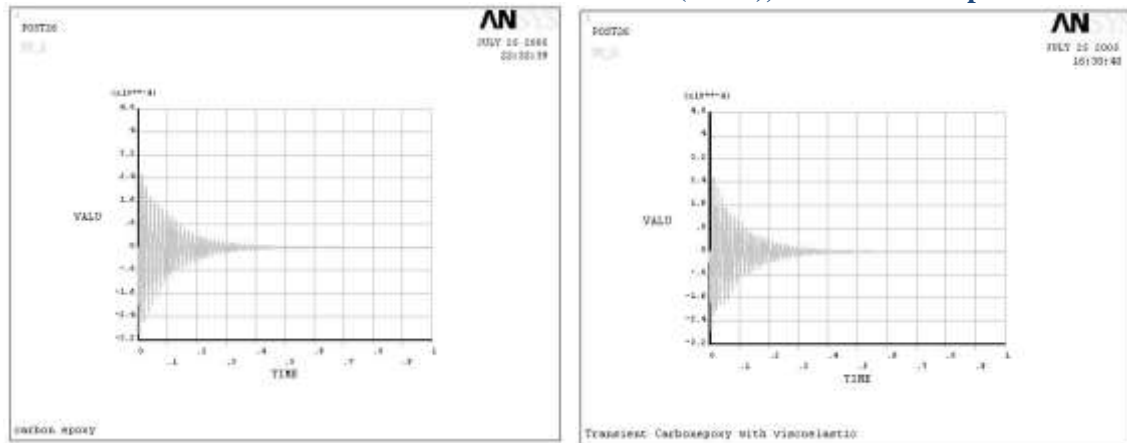


Fig 7.4 Transient Analysis for Carbon Epoxy Shaft **Fig 7.5 Transient Analysis for Carbon Epoxy Shaft with Rubber**

The Transient Dynamic Analysis of Carbon Epoxy Shaft using Shell 99 is shown in the Fig 7.4. From the above Fig 7.4 damping factor is calculated using Log Decrement method and the value is 0.02657. The Transient Dynamic Analysis of Carbon Epoxy Shaft using Shell 99 with Rubber is shown in the Fig 7.5. From the above Fig 7.5 damping factor is calculated using Log Decrement method and the value is 0.029005.

Optimization Of Damping Properties

It is noted that means proposed for improving free layer treatment, and the lightweight of fibers will benefit applications where weight is an important consideration. Fiber aspect ratio, fiber tip spacing, fiber angle and a number of other parameters were varied to improve the damping performance of a structural composite material. In this chapter, a micro mechanical model for the fiber-enhanced viscoelastic-damping polymer is established, and closed form expressions for the effective complex moduli are shown. Based on this model, damping performance of the enhanced damping polymer is optimized by establishing an optimal relation between the design parameters, such as length.

Viscoelastic Damping Treatment

Viscoelastic damping treatment performances include the following:

1. High loss modulus viscoelastic damping materials
2. High extensional stiffness constraining layers
3. A multiplicity of layers
4. Optimal section length constraining layer segments

Due to the high elastic modulus, usually twice and three times as much as that of steel and aluminum, respectively high elastic modulus fibrous materials, for example Kevlar or graphite fiber, are desirable candidates for constraint materials in viscoelastic damping treatments. By embedding fiber into a viscoelastic polymer matrix to make a single composite damping product, all the features cited above can be readily achieved. The optimal segment lengths of the fibers can be controlled in fabricating processes, and the effective number of damping layers can be increased to a great extent without difficulty. Furthermore, the proposed fiber enhanced viscoelastic damping polymer can be conveniently installed on structural surfaces as a diameter, spacing, and Young's modulus of the fibers and shear modulus of the viscoelastic matrix.

Effective Complex Module

Based on the fundamental principles of constrained viscoelastic layer treatments, a packing geometry with staggered layers as shown in the fig. 1 is selected, as it makes efficient use of the materials.

In this Fig 8.1, the rods with square cross section and finite length represent fibers, which are embedded in a viscoelastic matrix. The square fiber cross section is used in the model for convenience. The spacing between neighboring fibers is assumed to be the same and the gaps between fiber ends are neglected in the present analysis.

The Fig 8.2 also show a selected representative volume element whose geometrical characteristics and stress strain relations are the same for any of such elements, regardless of the position in the composite material. The average stress and strain of the representative volume element therefore are same for any such elements and also the same as that of the entire composite material under a uniform loading.

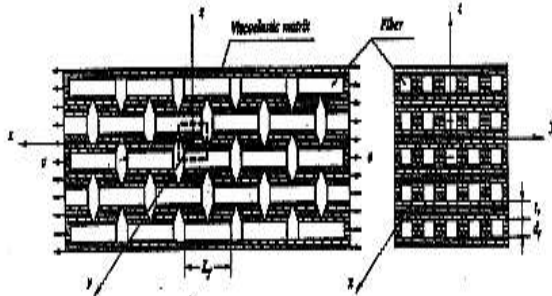


Fig 8.1

Fig 8.1 Schematic representation of the fiber enhanced viscoelastic polymer under stretching

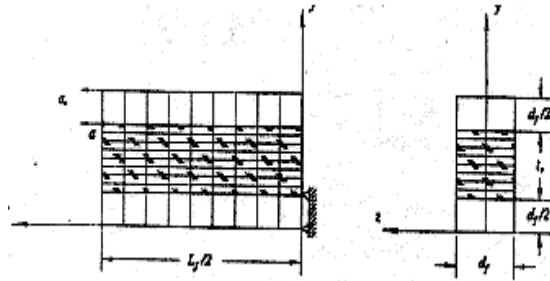


Fig 8.2

Fig 8.2 Schematic representation of finite element model for representative volume element

Displacement Field Under Axial Loading

Fiber enhanced Viscoelastic damping materials applied to the surface of flexible structures, will experience periodically axial loading when the base structure vibrates. In the development of the displacement field within the composite under this axial loading, the following assumptions are being made:

- Due to the much higher stiffness of the fibers, usually orders of magnitude higher than that of the viscoelastic matrix, it is assumed that the fiber sustains extensional stress only and the viscoelastic matrix transmits shear stress only.
- Uniform normal stress field and uniform shear stress field are assumed through the cross sections of the fibers and the viscoelastic matrix, respectively due to their very small dimensions.
- Linear material properties are assumed for the fibers and the viscoelastic matrix, as anticipated strains are small and well within the linear range.
- Poisson's ratio effects are negligible, again due to small strains.
- Fibers are elastic and dissipate very little energy relative to the viscoelastic matrix.

The front and backsides of the fibers are subjected to shear stresses due to existence of the viscoelastic matrix. Due to the symmetric configuration of the composite, the solution of the displacement field can be reduced to a two dimensional problem. It is well known that the complex modulus approach is an efficient method to describe the dynamic behavior of homogenous viscoelastic materials.

Based on the above assumptions and by applying boundary conditions, the final expressions for the effective storage modulus E_C' , loss modulus E_C'' and loss factor η_E the properties of the fiber enhanced viscoelastic damping polymers under uniaxial loading are given as:

$$E_C'' = E_f v_f \frac{N_2}{D} \dots\dots\dots (1)$$

$$E_C' = E_f v_f \frac{N_1}{D} \dots\dots\dots (2)$$

$$\eta_E = \frac{N_2}{N_1} \dots\dots\dots (3)$$

$$N_1 = (\coth \alpha)^2 \left(1 + \frac{\eta_v^2}{4} - \frac{\eta_v}{2\alpha} \cot \frac{\alpha \eta_v}{2} \right) + \left(\cot \frac{\alpha \eta_v}{2} \right)^2 \left(1 + \frac{\eta_v^2}{4} + \frac{1}{\alpha} \coth \alpha \right) + \frac{1}{\alpha} \left(\frac{\eta_v}{2} \cot \frac{\alpha \eta_v}{2} + \coth \alpha \right)$$

$$N_2 = \cot \frac{\eta_v}{2} \left[\frac{1}{\alpha} (\coth \alpha)^2 - 1 \right] + \frac{\eta_v}{2} \coth \alpha \left[1 + \frac{1}{\alpha} \left(\cot \frac{\alpha \eta_v}{2} \right)^2 \right] \quad (5)$$

$$D = (\coth \alpha)^2 \left[1 + \frac{\eta_v^2}{4} - \frac{\eta_v}{\alpha} \cot \frac{\alpha \eta_v}{2} + \frac{1}{\alpha^2} \left(\cot \frac{\alpha \eta_v}{2} \right)^2 \right] + \left[\cot \frac{\alpha \eta_v}{2} \right]^2 \left[1 + \frac{\eta_v^2}{4} + \frac{2}{\alpha} \coth \alpha \right] + \frac{2}{\alpha} \coth \alpha + \frac{\eta_v}{2} \cot \frac{\alpha \eta_v}{2} + \frac{1}{\alpha^2} \quad \dots \dots \dots (7)$$

$$\alpha = \frac{L_f \sqrt{G'_v}}{\sqrt{2t_v d_f E_f}}$$

Where,

Here, α represents the characteristic value, η_E represents the matrix loss factor, and V_f represents the fibre volume fraction. It can be seen that all these effective properties are functions of the packing geometry of the composite, and the material properties of the constituents. These properties will be employed in the analysis, which follows.

Table 8.1 Design Specifications

Sl.No	Quantity	Symbol	Unit	Value
1.	Distance between fibers	t_v	in	0.00021
2.	Diameter of fibers	d_f	in	0.0005
3.	Half length of fibers	$L_f / 2$	in	0.05
4.	Normal stress	σ	lb/in ²	1.44 e ⁵
5.	Shear modulus of matrix	G'_v	lb/in ²	130
6.	Tensile modulus of fibers	E_f	lb/in ²	6.4 e ⁷

Table 8.2 Calculated Values for $\eta_v=0.75$

α	N_1	N_2	D	$E''_C / V_f E_f$	η_E
0.2	4127.72	3379.17	121707.4	0.02776	0.81865
0.4	315.71	216.98	2475.22	0.08766	0.68726
0.6	81.34	43.79	323.58	0.13534	0.5384
0.8	34.14	13.87	92.37	0.1502	0.40636
1.0	18.43	5.51	39.65	0.13896	0.29902
1.2	11.53	2.47	21.51	0.11478	0.2141
1.4	7.94	1.17	13.47	0.087	0.14764

Table 8.3 Calculated Values for $\eta_v=1$

α	N_1	N_2	D	$E''_C / V_f E_f$	η_E
0.2	2083.28	2529.94	67781.21	0.03733	1.2144
0.4	169.24	161.88	1359.25	0.11909	0.95651
0.6	46.29	32.59	178.6	0.1825	0.7042
0.8	20.37	10.36	52.29	0.19808	0.50858
1.0	11.39	4.18	23.22	0.18009	0.36703
1.2	7.34	1.96	13.03	0.15026	0.26694
1.4	5.18	1.03	8.44	0.12182	0.19831

Table 8.4 Calculated Values for $\eta_v = 1.25$

α	N_1	N_2	D	$E_c''/N_f E_f$	η_E
0.2	1140	2019.39	42824.99	0.04715	1.77139
0.4	102.19	128.62	843.7	0.15245	1.25867
0.6	30.4	25.82	111.94	0.23068	0.84939
0.8	14.19	8.24	34	0.24239	0.58083
1.0	8.28	3.4	15.8	0.21527	0.41085

OPTIMIZATION OF DAMPING PROPERTIES

The closed form expressions of the effective material properties developed above allow optimization of the damping properties for the fiber enhanced viscoelastic polymers. The following discussion considers the establishment of the appropriate criterion for damping design optimization.

Intuitively, a large value of loss factor is a desired property for damping materials. Consider the damping mechanism in a viscoelastic material. In order to dissipate energy, a damping material must store some energy first. The more energy it can store, the more energy it will dissipate. The ability to store energy is measured by the storage modulus of materials. It is clear that the best quantity reflecting the damping performance of a viscoelastic material is the loss modulus of the material, i.e., the product of the loss factor and the storage modulus.

It may not always be desirable, though, to use a very high storage modulus to achieve a high loss modulus. The reason is that a high storage modulus of a damping material could significantly change the stiffness of the base structure on which it will be bonded. Therefore, a good design involves proper balance between loss factor and storage modulus.

The proposed strategy for optimization of the fiber-enhanced viscoelastic-damping polymer is given below.

- Examine the variation of the composite loss factor, η_E , with the characteristic value α .
- Based on the results of the step one, select a narrowed range of α to maximize the loss modulus, E_c'' .
- After all the parameters are determined, evaluate the storage modulus, η_E , and loss factor, E_c' .

The design specification for the model is shown in the table 8. 1. The calculated values are shown in table 8. 2, table 8. 3 and table 8. 4.

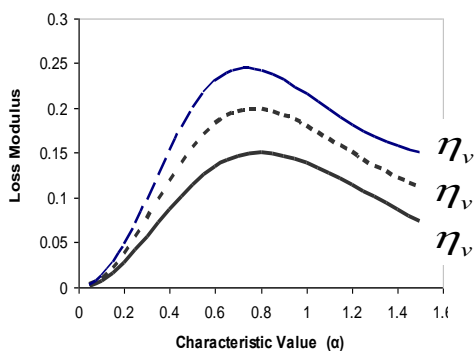


Fig 8.3 Effect of Characteristic value α on loss modulus ($E_c''/N_f E_f$)

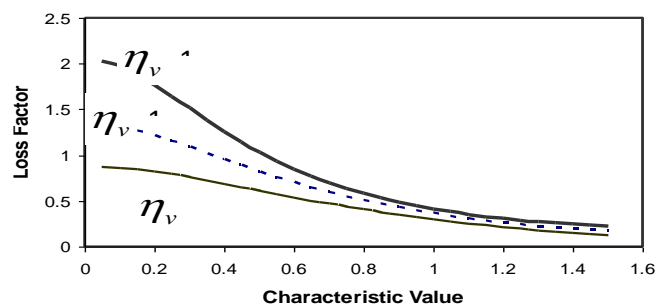


Fig 8.4 Effect of Characteristic value α on loss factor

DISCUSSIONS

For different values of matrix loss factors, η_v , maximum value of E_c'' occurs at the value of α around 0.75. It can be concluded that for maximum damping performance, the characteristic value of the composite, i.e., α , should be set to 0.75. To satisfy this optimum condition, one can vary the parameters used to calculate the characteristic value α .

(i.e., E_f , G_v' , t_v , d_f and L_f). For example, if E_f , G_v' , d_f and L_f are prescribed by a specific design, satisfaction of this condition will yield an optimal distance between fibers, t_v . A large value for the fiber Young's modulus E_f , is a desired property for the fiber enhanced viscoelastic damping polymer.

CONCLUSIONS

- The damping factor has been found out for Steel Shaft, Carbon Epoxy Shaft with and without Viscoelastic polymer (Rubber).
- The Static, Modal and Transient Dynamic Analyses have been carried out using Finite Element Analysis.
- The following observations were made by embedding the Viscoelastic polymer (Rubber) into the structure.
 - Damping factor increased by 16.3%, 9.2% for Steel, Carbon Epoxy shafts respectively.
 - The fundamental natural frequency increased by 12.76%, 7.2% for Steel, Carbon Epoxy shafts respectively.
 - The deflection value decreased by 21.38%, 12.71% for Steel, Carbon Epoxy Shafts respectively.
 - The increase in damping factor results in further suppression of vibrations and hence results in increased structural life.
- An optimal relation between design parameters such as the length, diameter, spacing, and Young's modulus of fibers and the shear modulus of viscoelastic matrix has been derived for achieving maximum damping performance. It has been found that for maximum damping performance, the characteristic value of the composite should be set to 0.75.

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